

# Vacuum-Structure and a Relativistic Pilot Wave

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## Abstract

We study a model for analyzing the effect of a principal violation of the Lorentz-invariance on the structure of vacuum. The model is based on the divergence theory developed by Salehi (1997). It is shown that the divergence theory can be used to model an ensemble of particles. The ensemble is characterized by the condition that its members are basically at rest in the rest frame of a preferred inertial observer in vacuum. In this way we find a direct dynamical interplay between a particle and its associated ensemble. We show that this effect can be understood in terms of the interaction of a particle with a relativistic pilot wave through an associated quantum potential.

## 1 Introduction

The requirement of the Lorentz-invariance is one of the basic foundations of modern physical theories. This principle states that all inertial frames of reference are equivalent. A key feature of the Lorentz-invariance would be its violation. Typically, such a violation would single out an individual frame in which a preferred inertial observer is at rest. The four-velocity of this preferred inertial observer may be interpreted as a time-like vector field which has almost the same value throughout the Einstein-Minkowski space. Such an absolute object in vacuum, which is an example of what is usually known as the internal vector  $N_\mu$  (Blokhintsev, 1964; Nielsen and Picek, 1983; Phillips, 1965) may generally be considered to constitute one of the basic characteristics of the violation of Lorentz-invariance.

Another physical characteristic of a Lorentz-noninvariant vacuum is provided by the concept

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of the universal length  $l_0$  (Blokhintsev, 1964; Nielsen and Picek, 1983) for which we adopt the following interpretation: The length  $l_0$  is used to define a minimal limiting length for all physical distances probed in 'typical' measurements. By a typical measurement we mean one which can be performed relative to a large (ideally infinite) number of physically equivalent inertial frames. Clearly, the length  $l_0$  acts as an absolute demarcation line between macroscopic (large) distances and microscopic (small) ones. It refers to an absolute property of a Lorentz-noninvariant vacuum.

Such possible limitations on the requirement of the exact Lorentz-invariance in vacuum, as reflected in the universal length  $l_0$  and the associated internal vector  $N_\mu$ , lead to immediate limitations on the applicability of the standard form of relativistic motion of a free particle. In fact, if the Compton wave-length (the typical size) of a free particle in vacuum approaches the universal length  $l_0$  we expect that the number of real systems which could act as equivalent inertial systems characteristic of the relativistic motion of the particle would reduce drastically. Specifically, we expect that the characteristic rest frame of a free particle with a Compton wave length  $\sim l_0$  to correspond to the rest frame of the preferred inertial observer characterized by the internal vector  $N_\mu$ . Thus, for such a particle a strong alteration of the standard form of the relativistic motion predicted by special relativity could in principle arise. The purpose of this paper is a consideration of this issue.

The organization of the paper is as follows. Section 2 provides a formal scheme for incorporating the invariance breaking effect of the universal length and the associated internal vector into the structure of vacuum in Einstein-Minkowski space. This scheme is presented in terms of the divergence theory developed by Salehi (1997). The divergence theory admits the treatment of some general constraints imposed on the vacuum by the condition of Lorentz-noninvariance. Also, we shall impose a boundary condition on the configuration of vacuum by combining the divergence theory with a time-asymmetric law discussed by Salehi and Sepangi (1999). Section 3 deals with the particle interpretation of the theory. The particles associated with the divergence theory may be interpreted as an ensemble of free particles with Compton wave-lengths having the order of magnitude of the universal length. This ensemble arises as a particular solution of the divergence theory and is found to be characterized by the condition that its rest frame corresponds to the rest frame of the preferred inertial observer characterized by the internal vector. It appears that the type of the particle motion within the ensemble is strongly constrained by this condition. Actually we find a dynamical interplay between an individual particle and its associated ensemble through a quantum potential generated by a pilot wave which we shall finally derive in the section 4.

## 2 Divergence theory

In order to study the violation of Lorentz-invariance in vacuum, the basic idea is to consider a dynamical coupling of a real scalar field  $\phi$  with the internal vector  $N_\mu$  in Einstein-Minkowski space. Following (Salehi, 1997) we start with the consideration of the current

$$J_\mu(\phi) = -\frac{1}{2}\phi \overset{\leftrightarrow}{\partial}_\mu \phi^{-1}. \quad (1)$$

It is easy to show that

$$J_\mu(\phi)J^\mu(\phi) = \phi^{-2}\partial_\mu\phi\partial^\mu\phi \quad (2)$$

and

$$\partial_\mu J^\mu(\phi) = \phi^{-1}(\Box\phi - \phi^{-1}\partial_\mu\phi\partial^\mu\phi) \quad (3)$$

in which,  $\Box = \eta_{\mu\nu}\partial^\mu\partial^\nu$  denotes the d'Alembertian associated with the Minkowski-metric  $\eta_{\mu\nu}$ . Combining the relations (2) and (3) we get the identity

$$\Box\phi - [J_\mu(\phi)J^\mu(\phi) + \partial_\mu J^\mu(\phi)]\phi = 0. \quad (4)$$

It should be noted that, this identity is a consequence of the definition (1) and not a dynamical law for  $\phi$ . However, one can obtain several dynamical theories by making various assumptions about the source of the divergence  $\partial_\mu J^\mu(\phi)$  in (4).

We analyze a model in which the source of the divergence is specified as (Salehi and Sepangi, 1999)

$$\partial_\mu J^\mu(\phi) = l_0^{-1}N_\mu J^\mu(\phi). \quad (5)$$

The corresponding field equation is

$$\Box\phi - [J_\mu(\phi)J^\mu(\phi) + l_0^{-1}N_\mu J^\mu(\phi)]\phi = 0. \quad (6)$$

Under the duality transformation

$$(\phi, N_\mu, \eta_{\mu\nu}) \rightarrow (\phi^{-1}, -N_\mu, \bar{\eta}_{\mu\nu}) \quad (7)$$

in which  $\bar{\eta}_{\mu\nu}$  is given by

$$\bar{\eta}_{\mu\nu} = \eta_{\mu\nu} + 2N_\mu N_\nu$$

equation (6) transforms to

$$\Box\phi - [J_\mu(\phi)J^\mu(\phi) + l_0^{-1}N_\mu J^\mu(\phi) - 2N_\mu N_\nu \partial^\nu J^\mu(\phi)]\phi = 0.$$

This equation can be rewritten as

$$\Box\phi - [J_\mu(\phi)J^\mu(\phi) + \partial_\mu J^\mu(\phi) - 2l_0 \frac{d}{d\tau}(\partial_\mu J^\mu(\phi))]\phi = 0 \quad (8)$$

where we have used

$$N_\nu \partial^\nu = \frac{d}{d\tau}$$

which is the defining relation of what we call the internal time-parameter  $\tau$  associated with  $N_\mu$ . Comparison of the last two terms in the brackets in equation (8) indicates that the physical effect of the last term is smaller than its previous term as a result of smallness of the universal length  $l_0$ . Therefore, the last term may be neglected<sup>§</sup> and equation (8) reduces to equation (6).

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<sup>§</sup>This assumption has been implicitly considered by Salehi and Sepangi (1999).

As a result, equation (6) becomes invariant under the duality transformation (7). The meaning of this duality is that, if we start from the configuration  $(\phi, N_\mu, \eta_{\mu\nu})$  on a Lorentzian domain (a domain in Einstein-Minkowski space with the metric  $\eta_{\mu\nu}$  and signature  $(-+++)$ ) we get another configuration by the duality transformation, namely  $(\phi^{-1}, -N_\mu, \bar{\eta}_{\mu\nu})$ , on an Euclidean domain (with the metric  $\bar{\eta}_{\mu\nu}$  and signature  $(++++)$ ). Therefore the duality transformation connects equivalent configurations of vacuum with different signatures.

To fix the causal structure of the Einstein-Minkowski space for our analysis, it is necessary to combine equation (5) with a duality breaking condition. On a Lorentzian domain we adopt the time-asymmetric condition (Salehi and Sepangi, 1999)

$$\partial_\mu J^\mu(\phi) = l_0^{-1} N_\mu J^\mu(\phi) > 0. \quad (9)$$

Clearly this condition can not simultaneously be satisfied for configurations related by the duality transformation, because the source of the divergence in (9) will reverse the sign under that transformation. Therefore, the condition (9) implies an assertion about both a preferred arrow of time, namely that determined by the internal vector  $N_\mu$ , and a preferred configuration of the scalar field  $\phi$ .

Now we proceed to derive a general constraint imposed on the configuration of the scalar field  $\phi$  by the condition of the duality breaking. If one combines the divergence law (5) with (1), one finds

$$\partial_\mu (J^\mu(\phi) - l_0^{-1} N^\mu \ln \phi) = 0.$$

We focus here on a particularly simple solution of this equation, given by the time-like current

$$J_\mu(\phi) = l_0^{-1} N_\mu \ln \phi. \quad (10)$$

Using this solution, we may then rewrite the divergence law (5) as

$$\partial_\mu J^\mu(\phi) = -l_0^{-2} \ln \phi.$$

Now, to conform with the duality breaking condition (9), one should require

$$\phi(x) < 1 \quad (11)$$

which constraints the configuration of the scalar field  $\phi$ .

### 3 The relativistic ensemble of particles

The divergence theory (5) was thought as a method for studying the effect of the Lorentz-noninvariance on the structure of the vacuum. Now, our purpose is to demonstrate that it involves certain features that can be interpreted in terms of an associated ensemble of particles. The key idea is to take the current  $J_\mu(\phi)$  as the defining characteristic of a fictitious ensemble. In specific terms, a particle in the ensemble that may potentially pass through a given point is assumed to have a four-momentum characterized by  $J_\mu(\phi)$ . Because of (10), such a particle would be absolutely at rest in the rest frame of the preferred inertial observer characterized by

the internal vector.

It is possible to convert the quantitative description of the ensemble into a more standard form. In fact, combining (6) with (1) and (10), we can derive

$$\partial_\mu S \partial^\mu S = l_0^{-2} \ln \phi + \left( \frac{\square \phi}{\phi} \right) \quad (12)$$

with  $S = \ln \phi + \alpha$  where  $\alpha$  is a constant. All terms in this equation are form-invariant under the scale transformation  $\phi \rightarrow \lambda \phi$ ,  $\lambda$  being an arbitrary constant, except the first term on the right hand side. Thus, this term is sensitive to the background average value  $\bar{\phi}$  of the scalar field  $\phi$ . We may, therefore, use a background approximation for this term, namely by replacing the  $\ln \phi$  by  $\ln \bar{\phi}$  which should be negative due to (11). We shall subject the choice of  $\bar{\phi}$  to the condition that  $\ln \bar{\phi}$  shall adjust  $l_0^{-2} \ln \bar{\phi}$  to the characteristic mass-scale  $l_0^{-1}$  of the theory, leading to

$$\partial_\mu S \partial^\mu S = -M^2 + \left( \frac{\square \phi}{\phi} \right) \quad (13)$$

with  $M^2 = -l_0^{-2} \ln \bar{\phi} \approx l_0^{-2}$ . Equation (13) can be interpreted as the Hamilton-Jacobi equation characterizing the ensemble. In this ensemble the particle-mass  $M$  is adjusted to  $l_0^{-1}$  by a corresponding background average value of the scalar field. Furthermore, the term  $\frac{\square \phi}{\phi}$  indicates a modification of the particle mass due to a local deviation of the scalar field from its background average value. This term reflects a direct interplay between a particle and its associated ensemble.

We should emphasize the distinguishing feature of equation (13). The dynamical properties of the particles in the ensemble (the configuration of the S-function up to an additive constant) basically emerges as a consequence of a local deviation of the scalar field from its background average value  $\bar{\phi}$ . Moreover, the latter value serves to adjust the mass of the particle in the ensemble to the characteristic mass-scale of the theory, namely  $l_0^{-1}$ . In this way the scalar field  $\phi$  is found to carry the whole dynamical characteristics of an ensemble of particles. Now, we try to characterize the ensemble in terms of a hydrodynamical equation. To derive such an equation we begin with

$$\begin{aligned} \partial_\mu (\phi^2 J^\mu(\phi)) &= 2\phi \partial_\mu \phi J^\mu(\phi) + \phi^2 l_0^{-1} N_\mu J^\mu(\phi) \\ &= 2\phi^2 J_\mu(\phi) J^\mu(\phi) + \phi^2 l_0^{-1} N_\mu J^\mu(\phi). \end{aligned}$$

To rewrite this equation in a convenient form, we linearize the quadratic term in  $J_\mu(\phi)$  on the right hand side to obtain the approximate relation

$$\partial_\mu (\phi^2 J^\mu(\phi)) \approx 2\phi^2 \bar{J}_\mu(\phi) J^\mu(\phi) + \phi^2 l_0^{-1} N_\mu J^\mu(\phi).$$

Here the constant current  $\bar{J}_\mu(\phi)$  may be obtained from (10) by replacing  $\phi$  by its corresponding background average value of  $\bar{\phi}$ , used previously in the derivation of equation (13) from equation (12). Actually we get  $\bar{J}_\mu(\phi) \approx -l_0^{-1} N_\mu$ , leading to

$$\partial_\mu (\phi^2 J^\mu(\phi)) \approx -\phi^2 l_0^{-1} N_\mu J^\mu(\phi). \quad (14)$$

Using the definition (1), equation (14) can be brought into the form

$$\partial_\mu (\phi^2 \partial^\mu S) \approx -\phi^2 l_0^{-1} \frac{d}{d\tau} \ln \phi. \quad (15)$$

The right hand side of this equation, which in principle may be different from zero, indicates that, if we relate  $\phi^2$  to the particle-number density, the conservation of particles in the ensemble may be violated.

## 4 Derivation of the pilot wave

It is possible to interpret the equation of motion for a particle in the ensemble in terms of a pilot wave. Consider the wave

$$\psi = \phi e^{iS}$$

which satisfies, as a consequence of equations (13) and (15), the equation

$$\square\psi - M^2\psi \approx -(il_0^{-1} \frac{d}{d\tau} \ln |\psi|)\psi.$$

This equation reduces to the relativistic Klein-Gordon equation in the idealized limit in which any conceivable dependence of  $\psi$  on the internal time-parameter  $\tau$  is ignored.

The merit of introducing the wave  $\psi$  is that it acts as a sort of a pilot wave in the sense of causal interpretation of quantum mechanics (Holland, 1993; Bohm, 1952; Bohm and Hiley, 1993), the term  $\frac{\square\phi}{\phi}$  on the right hand side of equation (12) being the associated quantum potential.

It should be emphasized that the pilot wave derived in this section is thought to mediate merely the interaction of a particle with its associated ensemble. The necessity of this interaction is deeply connected with the fact that the characteristic rest frame of the ensemble at every point appears to be just the rest frame of the preferred inertial observer characterized by the internal vector, as reflected in (10). The corresponding quantum potential just serves to incorporate this common global memory effect. We should note that this behavior does not seem to be symptomatic of any relativistic particle. In fact, if the Compton wave-length of a relativistic particle is much larger than the universal length  $l_0$ , we generally expect to find a large number of real systems as equivalent inertial systems characteristic of the relativistic motion. However, this number may reduce drastically as the mass of a relativistic particle approaches the mass-scale  $l_0^{-1}$ . One may ask how this expected universal scaling behavior common to all relativistic particles is related to their quantum behavior. The exploration of this question is an interesting subject which requires considerable clarification.

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